MATHEMATICAL FORMULAE AND PHYSICAL CONSTANTS

Calculus

$f(x)$	$f'(x)$
x^n	nx^{n-1}
e^x	e^x
$\log_e x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sin^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2 - x^2}}$
$\cos^{-1}\left(\frac{x}{a}\right)$	$-\frac{1}{\sqrt{a^2 - x^2}}$
$\tan^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{a^2 + x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\sinh^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{x^2 + a^2}}$
$\cosh^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{x^2 + a^2}}$
$\tanh^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{a^2 - x^2}$
uv	$u'v + uv'$
$\frac{u}{v}$	$\frac{u'v - uv'}{v^2}$

Definite Integrals

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}} (n \geq 0)$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$

Integration by Parts

$$\int u \frac{dv}{dx} dx = uv - \int \frac{du}{dx} v dx$$

Spherical Polar Coordinates

$x = r \sin\theta \cos\phi \qquad y = r \sin\theta \sin\phi \qquad z = r\cos\theta \qquad dV = r^2 \sin\theta dr d\theta d\phi$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

Physical Constants

charge on electron	$e = 1.60 \times 10^{-19}$ C
mass of electron	$m_e = 9.11 \times 10^{-31}$ kg = 0.511 MeV/c^2
mass of proton	$m_p = 1.673 \times 10^{-27}$ kg = 938.3 MeV/c^2
mass of neutron	$m_n = 1.675 \times 10^{-27}$ kg = 939.6 MeV/c^2
Planck constant	$h = 6.63 \times 10^{-34}$ J s
(Planck constant)/2π	$\hbar = 1.05 \times 10^{-34}$ J s
Boltzman constant	$k = 1.38 \times 10^{-23}$ J K^{-1}
	$= 8.62 \times 10^{-5}$ eV K^{-1}
speed of light in free space	$c = 3.00 \times 10^{8}$ m s^{-1}
permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
Avogadro constant	$N = 6.02 \times 10^{23}$ (g-mol)$^{-1}$
gas constant	$R = 8.32$ J (g-mol)$^{-1}$ K^{-1}
ideal gas volume (STP)	$V_0 = 22.4\ l$ (g-mol)$^{-1}$
gravitational constant	$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
Rydberg constant	$R_\infty = 1.10 \times 10^{7}$ m^{-1}
Bohr radius	$a_0 = 0.529 \times 10^{-10}$ m
Bohr magneton	$\mu_B = 9.27 \times 10^{-24}$ J T^{-1}
fine structure constant	$\alpha \approx 1/137$
Wien displacement law constant	$b = 2.898 \times 10^{-3}$ m K
Stefan constant	$\sigma = 5.67 \times 10^{-8}$ Wm^{-2} K^{-4}
radiation density constant	$a = 7.55 \times 10^{-16}$ J m^{-3} K^{-4}
mass of sun	$M = 1.99 \times 10^{30}$ kg
radius of sun	$R = 6.96 \times 10^{8}$ m
luminosity of sun	$L = 3.85 \times 10^{26}$ W
mass of earth	$M_\oplus = 6.0 \times 10^{24}$ kg
radius of earth	$R_\oplus = 6.4 \times 10^{6}$ m

Conversion Factors

1 u (atomic mass unit)	$= 1.66 \times 10^{-27}$ kg
	= 931.5 MeV/c^2
1 Å (angstrom) = 10^{-10} m	1 g (gravity) = 9.81 m s^{-2}
1 eV = 1.60×10^{-19} J	
1 atmosphere = 1.01×10^{5} Pa	1 year = 3.16×10^{7} s
1 parsec = 3.08×10^{16} m	1 astronomical unit = 1.50×10^{11}m

Binomial Expansion

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \ldots + \binom{n}{k}x^k y^{n-k} + \ldots + y^n$$

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots (|x| < 1)$$

Plane Polar Coordinates

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$dA = r dr d\theta$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$
$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$
$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$
$$\cos A - \cos B = 2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Series Expansions

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$
$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots (|x| < 1)$$

Taylor's Series

$$f(x+a) = f(a) + xf'(a) + \frac{x^2}{2!} f''(a) + \ldots$$

Spherical Geometry

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$
$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

Vectors

$$\underline{A}.\underline{B} = A_x B_x + A_y B_y + A_z B_z$$
$$\underline{A} \times \underline{B} = (A_y B_z - A_z B_y)\underline{i} + (A_z B_x - A_x B_z)\underline{j} + (A_x B_y - A_y B_x)\underline{k}$$
$$\underline{A} \times (\underline{B} \times \underline{C}) = (\underline{A}.\underline{C})\underline{B} - (\underline{A}.\underline{B})\underline{C}$$
$$\underline{A}.(\underline{B} \times \underline{C}) = \underline{B}.(\underline{C} \times \underline{A}) = \underline{C}.(\underline{A} \times \underline{B})$$

Vector Calculus

$$\nabla\phi = \frac{\partial\phi}{\partial x}\underline{i} + \frac{\partial\phi}{\partial y}\underline{j} + \frac{\partial\phi}{\partial z}\underline{k}$$
$$\nabla.\underline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$
$$\nabla \times \underline{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\underline{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\underline{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\underline{k}$$
$$\nabla.(\nabla\phi) = \nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$
$$\nabla \times (\nabla\phi) = 0$$
$$\nabla.(\nabla \times \underline{A}) = 0$$
$$\nabla \times (\nabla \times \underline{A}) = \nabla(\nabla.\underline{A}) - \nabla^2 \underline{A}$$

Stirling's Formula

$$\log_e N! \approx N \log_e N - N$$

Series

$$\sum_{k=1}^{n} [a + (k-1)d] = \frac{n}{2}[2a + (n-1)d]$$
$$\sum_{k=1}^{n} ar^{k-1} = a\frac{(1-r^n)}{1-r}$$

Exponentials

$$e^{i\theta} = \cos\theta + i\sin\theta$$
$$\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$
$$\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$$
$$\cosh\theta = \frac{1}{2}(e^{\theta} + e^{-\theta})$$
$$\sinh\theta = \frac{1}{2}(e^{\theta} - e^{-\theta})$$